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Reason I

Understanding, insight, and intellectual power

I see the green field and I believe that it is there before me. I look away, and I believe that I am now imaging it. I remember its shape, and I believe that it is rectangular. These are beliefs grounded in my experience: perceptual, self-conscious, and memorial. But I also believe something quite different: that *if* the spruce to my left is taller than the maple to my right, then the maple is shorter than the spruce.

On what basis does one believe this obvious truth? Do we even need to see the trees to know it? Certainly it is on the basis of perception that I believe *each* of the two comparative propositions; it is easy to see, for instance, that the spruce is Reason I 2 taller than the maple. But I do not believe on the basis of perception that *if* the spruce is taller than the maple then the maple is shorter than the spruce. As a rational being, I quite easily grasp this truth and thereby believe it.

The kind of apparently elementary use of reason this case illustrates seems basic for both knowledge and justification. But there is continuing debate about the nature and grounds of our knowledge and justification regarding the simple, obvious truths that we seem to know just in virtue of the kind of understanding of them any rational being might be expected to have. A good way to understand the epistemological role of reason is to begin with a notion that seems central for the most basic kind of knowledge and justification reason gives us—*self-evidence*.

[*1. What is self-evidence? Show its role in knowledge and justification.* সেলফ-এভিডেন্স কী? জ্ঞান ও যাচাইকরণে এর ভূমিকা নির্দেশ করো।]

Self-evident truths of reason

Such truths as the luminous one that if the spruce is taller than the maple then the maple is shorter than the spruce have been said to be evident to reason, conceived roughly as a mental capacity of understanding. They are presumably called *selfevident* because they are thought to be evidently true taken by themselves, with no need for supporting evidence. Indeed, they are often considered *obvious* in themselves, roughly in the sense that simply upon attentively coming to understand them, one normally sees their truth and thereby knows them.

The concept of self-evidence

In the light of such points, we might more specifically characterize selfevident propositions as those truths such that (1) if one (adequately) understands them, then by virtue of that understanding one is justified in (hence has justification for) believing them, and (2) if one believes them on the basis of (adequately) understanding them, then one thereby knows them.[1] (1) says roughly that understanding them suffices for being situationally justified in believing them; it provides a justification for belief—which one can have without actually believing the proposition in question. (2) says in effect that this understanding can ground knowledge: the understanding is sufficient to render a belief based on it knowledge. (2) implies, then, that self-evident propositions are true. This implication is appropriate, since the self-evident is standardly regarded as true (and for clarity I have put truth explicitly into the characterization above).

What I have said does not imply, however, that the kind of justification one gains from understanding the self-evident is *indefeasible*, that is, so secure that it cannot be defeated, rather than prima facie. If the understanding in question is eliminated or obscured, the belief may cease to be justified. But at least some cases of this kind of justification are plausibly taken to exhibit justification as strong as any we can have. It can be difficult to see how defeasibility can occur here because self-evident truths are so commonly considered *also* obvious. But not all of them are—at least to finite minds. Apart from logical training, certain self-evident logical truths are not obvious to us; and it may not be obvious to most Reason I 3 of us, on first considering it, that first cousins share a pair of grandparents. But this satisfies both (1) and (2) and is self-evident.[2]

There is an important analogy to perception. Just as one can see a visible property of something, such as its rectangularity, without believing that it has that property, one can comprehendingly (understandingly) consider a self-evident proposition without coming to believe that proposition; and just as one's seeing a bird fly past gives one justification for believing it did whether or not one forms this belief, adequately understanding the proposition that if the spruce is taller than the maple, the maple is shorter than the spruce, gives one (situational) justification for believing this whether one does or not.

When it comes to concepts, there is a further analogy to perception: a hierarchy analogous to the perceptual one. There is understanding a concept, such as *being taller than*. Second, there is objectually believing it to apply to something, say to a pair of things, such as the spruce and the maple. Third, there is propositionally believing something that "applies it," as when one conceives the trees as, say, the spruce and the maple, and believes that the spruce is the taller.[3]

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[1] *Adequacy* of understanding of a proposition cannot be merely partial understanding, and it is more than simply getting the general sense of a sentence expressing it, as where one can analyze the grammar of the sentence, indicate something of what it means through examples, and perhaps translate it into another language one knows well. Adequacy here implies not only seeing what the proposition says but also being able to apply it to (and withhold its application from) an appropriately wide range of cases. This matter is treated in some detail in my 'Self Evidence', *Philosophical Perspectives* 13 (1999), 205–28. Note also that there is no appeal here to understanding or positing the *necessity* of the propositions (though the characterization lends itself to taking them to be necessary). In this respect my notion of the self-evident is simpler and more moderate than the traditional one common in much of the literature. See, for example, Laurence BonJour, 'Toward a Moderate Rationalism', *Philosophical Topics* 23, 1 (1995), 47–78, esp. section 3.

[2] For a helpful discussion of obviousness related to (but quite different from) the one in my 'Self-Evidence' and connected with the theory of the a priori in general, see Robin Jeshion, 'On the Obvious', *Philosophy and Phenomenological Research* 60, 2 (2000), 333–55.

[3] Two points are appropriate here. (1) A fourth case is one in which a concept is not only *exercised* in a belief but explicitly figures in it, as when one believes that the concept *being taller than* is instantiated by the spruce and the maple. (2) The analogy between perception and conception I am developing is meant to leave open what concepts are and what it is to understand one. As will later be apparent, philosophers differ in their

With self-evident propositions like the straightforward proposition that if the spruce is taller than the maple then the maple is shorter than the spruce, one need not consult one's experience of the kind of thing described, or even ponder the propositions in question, in order to grasp—roughly, to understand— those propositions. And <u>when one does come to understand them and focuses on them in the light of that understanding, one thereby normally comes to believe and know that they are true</u>.[4]

[**2. Are self-evidents indefeasible or prima-facie? Give your opinion in this regard.** সেলফ-এভিডেন্টগুলো কি ভ্রান্তির সম্ভাবণারহিত, না কি প্রাথমিক-প্রভাবণা মাত্র? এ বিষয়ে তোমার মত দাও]

Two types of immediacy

There are many truths which, as just illustrated, we readily grasp and thereby immediately believe. <u>In the simple case of comparison of heights, our belief is *immediate* in both (1) the temporal sense of 'instantly formed' and (2) the *epistemic sense*—the sense entailing that we see their truth without inferring them from anything else</u>. The point, then, is not the temporal one that we grasp them instantly, though we may. What is crucial is that our belief exhibits *epistemic immediacy*: the belief is not based on inference or on a further, evidential belief. If it were, it would be <u>epistemically mediate</u>: mediated by (and thereby at least partly grounded in) the set of premises from which we infer (or on the basis of which we believe) the proposition, as my belief that Socrates is mortal is mediated by the two propositions which are part of the basis of my believing this: that he is a human being, and that all human beings are mortal.[5]

The proposition that Socrates is mortal is in another way unlike the proposition that if the spruce is taller than the maple then the maple is shorter than the spruce. It is not self-evident. There are at least two ways to explain why. First, Socrates and mortality are not intrinsically connected, as are one thing's being taller than a second and the second's being shorter than the first. An omnipotent God could have kept him in existence. Second (and speaking more generally), to know that Socrates is mortal one needs more than reflection—a temporally extended use of reason—on this proposition. <u>One apparently needs information not given by the proposition</u>. Even thinking of him as a human being does not absolutely preclude every route to his immortality. But reflection—indeed even *intuition* as a sometimes momentary use of reason—indicates that the spruce's being taller than the maple precludes the maple's not being shorter than the spruce.

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understanding of the truths of reason in part because of their different understandings of the nature of concepts.

[4] One reason for the normality qualification is to make room for the possibility that one can consider and adequately understand a self-evident proposition yet fail to believe it. Brain manipulation might cause such failure. We should also make room for the possibility that, especially with more complex selfevident propositions—say that if p entails q and q entails r and r entails s, and s is not true, then p is false—it may take a person time to form the belief.

[5] *Temporal immediacy*, unlike epistemic immediacy, is a property not primarily of beliefs as such but of their formation. A belief is temporally immediate when its formation occurs "without delay" upon the person's considering the proposition in question (or encountering the situation, such as the sight of a lightening bolt, that gives rise to the belief). One could also say that propositions are temporally immediate in a derivative sense when they are so obvious that one normally believes them immediately on (comprehendingly) considering them. Many selfevident propositions are like this. But when I consider some self-evident propositions, such as that if there never have been siblings, then there never have been first cousins, it may or may not take me a moment to see their truth. Still, when one does see such a truth, the belief one forms will (at least normally) be epistemically immediate, not inferential. So, this proposition and my coming to believe it may or may not be temporally immediate. By contrast, the proposition that I am now seeing print is temporally immediate (for me) but is not self-evident. It is evident not in itself, but *through* what I see.

This kind of point concerning propositions such as the one about the two trees has led philosophers to consider them to be *truths of reason*—roughly, truths knowable through the use of reason as opposed to reliance on sense experience. This kind of knowability has led philosophers to regard them as also *necessarily true—necessary*, for short, that is, such that their falsehood is absolutely precluded: there are simply no circumstances in which they are false. If a proposition is not necessary (necessarily true) *and* its negation is also not necessary, it is called *contingent*, because whether it is true—that is, its truth *or* falsity, in another terminology—is contingent on (dependent on) circumstances. That there are more than two trees in my yard is contingent. There are more, but there need not be: the number is contingent on how many I want.

[*3. What are the two types of immediacy? Explain epistemic immediacy as the form of truths of reason.* দুই ধরনের অব্যবহিতা কী? বৌদ্ধিক সত্যতার প্রকরণ হিসাবে জ্ঞানতাত্ত্বিক অব্যবহিতা ব্যাখ্যা করা]

The classical view of the truths of reason

How might we understand the justification of our beliefs of self-evident and apparently necessary propositions? And how do we know them? The best known answers to these questions, and probably the only ones we should call the *classical* answers, derive largely from Immanuel Kant, though there are similar ideas in earlier philosophers who very likely influenced Kant. He discussed both the truth of the kinds of propositions in question and how we know them.[6]

What Kant said is complex and difficult to interpret precisely, and I am simply going to lay out a version of the classical account which may correspond only roughly to Kant's views. Moreover, although I am interested mainly in justification and knowledge regarding the truths of reason, I will also talk about the basis of these truths themselves when that is useful in discussing how we can know or justifiedly believe them.

Analytic propositions

Take the proposition that all vixens are female. I easily grasp its truth, and I immediately believe it: I depend on no premises or evidence. There was a time when 'vixen' was not in my vocabulary. I might then have looked at the sentence 'All vixens are female' and not known what proposition it expressed, much less seen the particular truth (true proposition) it does express. But this point does not show that I do not immediately believe that truth once I *do* (comprehendingly) consider it. It shows only that encountering a sentence which expresses a truth does not enable one to consider that truth unless one *understands* the sentence.

We can see, moreover, that when we do consider the truth that all vixens are female, we do not (or at least need not) know it on the basis of beliefs about the *sentence* 'All vixens are female'. For we can consider that same truth by using some other sentence to express it (say in Spanish), and perhaps without using a sentence at all.[7] If, however, we think about what grounds the truth of the

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[6] Kant's most detailed presentation of his views on these matters is in his *Critique of Pure Reason* (first published in 1781), but a short presentation is provided in the Preamble to his *Prolegomena to Any Future Metaphysics* (1783). Kant's conception of the analytic is quite reminiscent of Aquinas's idea that the self-evident has "its predicate contained in the notion of the subject" (*Summa Theologiae,* Question 94, Article 2).

[7] There has long been controversy about whether such thought is possible without using language, or at least having a language. Donald Davidson is among those to argue for a strong dependence of thought on language. See, for example, his *Inquiries into Truth and Interpretation* (Oxford: Oxford University Press, 1984). Relevant critical discussion of Davidson is provided by Ruth Barcan Marcus in 'Some Revisionary Puzzles about Belief and Believing', *Philosophy and Phenomenological Research*, supplement to vol. 50 (1990), 133–53, which brings out serious problems for the view that beliefs must have sentence-like objects. There is no need to take a stand on this issue for my main purposes in this book.

proposition, we may discover something which in turn helps to explain why we so readily understand and believe it.

To get a sense of the ground of this truth, consider what a vixen is. It is a female fox. Indeed, the concept of a vixen may be analyzed in terms of being female and being a fox. So, in saying that a vixen is a female fox, one could be giving an elementary analysis of the concept of a vixen. Now suppose that (like Kant) we think of an analysis of a concept as indicating what the concept *contains, or, in a certain way, includes*. We can now say that the concept of being female is part of the concept of a vixen, and that being female is thus an element in being a vixen.[8] In the light of all this, we might call the truth that all vixens are female an *analytic proposition*. To cite one major conception Kant presented, this is a proposition such that what it predicates of its subject can be "analyzed out of" the concept of that subject. Here the subject is vixens (or any arbitrarily given vixen), and the predicate is *being female*, which is part of, and so analyzable out of, the concept of a vixen. The same sort of thing holds for the propositions that all bachelors are unmarried, that all triangles have three angles, that all sound arguments have true premises and true conclusions, and so on. Analytic propositions are usually considered clear cases of the self-evident.[9] This is explainable in terms of our account of the self-evident—provided we make the not implausible assumption that, given an adequate understanding of ~~such~~ a proposition, one can frame an analysis in which the containment relation is clearly evident. By contrast, Kant and others have viewed nonanalytic propositions as *empirical*, and taken empirical propositions to be knowable not by using reason alone but only on the basis of confirmatory experience—most prominently (and perhaps necessarily) perception.

[**4. Does an analytic proposition mean as understanding a sentence or believing a sentence? Focus on the difference between an analytic statement and an empirical one in this regard.** বিশ্লেষক বচন বলতে একটি বাক্যকে অনুধাবন করা বুঝায়, না কি বিশ্বাস করা বুঝায়? এই সুবাদে বিশ্লেষক উক্তি ও অভিজ্ঞতাজাত উক্তির পার্থক্য তুলে ধরা]

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[8] *One* way to conceive this is as follows: if the concept of *F* is part of the concept of *G*, then having the property (of) *F* is self-evidently entailed by having the property (of) *G*. I do not accept this overall conception of conceptual containment but do believe that the entailment holds (even if not self-evidently).

[9] This is plausible if (1) the correct analysis of a key concept in an analytic proposition, say that of a vixen, is discoverable, without reliance on anything beyond understanding that concept, by anyone with an (adequate) understanding of the proposition, and (2) given a correct analysis of that concept, the truth of the analytic proposition is appropriately evident. However, some analytic propositions are not understandable in this way; some might be *provable* only by a lengthy process from one that is (a notion discussed on page 113). Further, it is by no means clear that every analytic proposition is self-evident in the very common sense that implies a fairly high degree of obviousness. If, as seems plausible, the self-evidence of a proposition simply implies that *some* kind of adequate understanding is sufficient for justification for believing it, then we might plausibly distinguish between the immediately and the mediately self-evident and allow that the latter propositions may be understandable (to normal persons) only on the basis of considerable reflection. Cf. Thomas Aquinas's view (which Kant might have known) that:

> Any proposition is said to be self-evident in itself, if its predicate is contained in the notion of its subject . . . *Man is a rational being*, is, in its very nature, self-evident, since he who says *man* says *a rational being;* and yet to one who does not know what a man is, this proposition is not self-evident . . . some propositions are self-evident only to the wise, who understand the meaning of the terms of the propositions.

(*Summa Theologiae,* Question 94, Article 2)

This seems to anticipate Kant's containment notion of the analytic and largely accords with the conception of the self-evident I have introduced.

Necessary propositions

This way of looking at our example helps to explain something else that is true of the proposition that all vixens are female: it *cannot* be false and, in that sense, is necessary (a necessary truth). To see this point, try to conceive of a non-female vixen. Since the concept of a vixen is analyzable as (and hence equivalent to) that of a female fox, one is in effect trying to conceive of a *non*-female female fox. This would be both female and not female. We would have a contradiction. Hence, there cannot be such a thing, on pain of contradiction. It is thus absolutely impossible—in a sense implying impossibility by the laws of logic—that there be a non-female vixen. By contrast, it is possible that there is, and also that there is not, a 200-pound vixen. The proposition that all vixens weigh less (or more) than this is *contingent*: neither necessarily true nor necessarily false.

Because the falsity of analytic propositions entails a contradiction in this way, they are often thought to be—and are sometimes even *defined* as—those that are true *on pain of contradiction*. That is, their falsity entails a contradiction, and hence they can be false only if a contradiction is true. That is absolutely impossible. Analytic propositions are therefore regarded as truths that hold in *any* possible situation and hence are necessary (though other kinds of truths are also considered necessary).

Now if analytic propositions are true by virtue of the sort of conceptual containment relation we have been exploring, might we not know each one we do know in virtue of grasping the containment relation basic to it, in the sense that we have an adequate understanding of that relation? In considering the proposition that all vixens are female, one in some way grasps the containment relation between the concept of a vixen and that of being female. Intellectually— *intuitively*, in one widely used terminology—one sees the relation and thereby sees and (non-inferentially) knows the truth it underlies.

It might be objected that the correct account is instead this. One quickly or subconsciously reasons: The concept of a vixen is analyzable as that of a female fox; *being female* is contained in that analysis; hence all vixens are female. So, it may be claimed, one knows that all vixens are female only inferentially. A defender of the classical view would reply that this second-order reasoning indicates how one might *show* that one knows that all vixens are female, but it does not indicate *how* one *knows* it, at least not if one just grasps its truth in the normal way.

The classical account can grant that one perhaps *could* come to know the proposition in that indirect way, by conceptual analysis. But one need not come to know it in that way; and normally, if one did not *already* know that vixens are female foxes, one would not even be in a *position* to know (on one's own) the sophisticated truth that the *concept* of a vixen is analyzable as that of a female fox. Believing that all vixens are female, in virtue of grasping the crucial containment relation between the concept of a vixen and that of a female, does not require coming to know this proposition in that sophisticated way.

[*5. Explain and evaluate the nature of containment relation in a necessary statement.* কোনো আবশ্যিক বচনের মধ্যকার অন্ত:সম্পর্কের ধরন ব্যাখ্যা ও মূল্যায়ণ করো।]

The analytic, the a priori, and the synthetic

We can now see how the classical account of the truths of reason might apply to apparently *non-*analytic truths that are directly and intuitively grasped. Think about the proposition that nothing is both red and green all over at one time (different kinds of examples will be considered in Chapters 12 and 14). This is apparently self-evident and hence a truth of reason. But is it analytic? Can we analyze *being non-red* out of the concept of being green, or *being non-green* out of the concept of being red, so that anyone who said that something is red and green all over at once could be shown to

be implying that it is (wholly) red and non-red, or green and non-green? This is doubtful. For one thing, it is not clear that we can analyze the concept of being red (or the concept of being green) *at all* in the relevant sense of 'analyze'. Still, on the classical view, we can know through the use of reason the necessary truth that nothing is red and green all over at once.

Let us consider two kinds of objections to the idea that the proposition that nothing is red and green all over at once is self-evident and necessary, yet not analytic. The first is based on treating the proposition as empirical and contingent; the second objection says it is analytic after all.

Take the contingency objection first. One might think that there could be a scientific explanation of why nothing is red and green all over at once; and if there is, then (on a plausible and standard view of such matters) the proposition is empirical and not self-evident or even necessary. How might such an explanation go? We can, after all, scientifically clarify what being red (or any other color) is by appeal to facts about light. This might seem to enable us to know all there is to know about basic relations among colors, even though the relevant facts about light are contingent. On the classical view, however, although scientific investigation helps us to understand certain facts about red *things* (and perhaps about the property of being red), it does not indicate what is essential to the *concept* of a red thing, such as being non-green at the time it is red. Similarly, it is essential to the concept of a vixen that it is equivalent to that of a female fox.

To be sure, one could discover scientifically that vixens have a unique tracking system. But normally one would be identifying them for study as *female* foxes and hence would not set out to discover whether they are female. On the classical view, we cannot identify anything as a vixen— say, for experimental purposes— except under the assumption that it is female. Thus, the possibility of discovering anything inconsistent with its being female is ruled out from the start. If our experimental subject is *selected* by its having a specified property, we cannot find out experimentally that *it* (as opposed to something else it may turn into) lacks that property.

Similarly, one would not normally set out to discover scientifically whether what is red all over is ever also green all over at the same time—since it would be at best difficult to wonder whether this is true without immediately *seeing* that it is. This does not make analytic or any self-evident truths more important than scientific truths. The former are simply different: they are not of the right kind to be open to scientific verification or falsification, and in part for this reason they also do not compete with scientific truths.

It appears, then, that the suggested "scientific" objection to the classical view fails. If, however, the proposition that nothing is red and green all over at once is not a "scientific truth," that might be because it is analytic after all. Let us explore further whether the classical view is correct in claiming that the two self-evident truths in question still differ in this: being non-green is not analyzable out of the concept of being red, whereas being female is analyzable out of the concept of being a vixen.[10]

This brings us to the second objection. The objection proceeds by arguing (against the classical view) that the proposition that nothing is red and green all over at once is analytic. Could one not *indirectly* analyze the concept of being red as equivalent to the concept of having a color *other than*

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[10] There are philosophers who regard colors as subjective in a way that might seem to undermine the example here. I do not see that taking the proposition that nothing is red and green all over at once to be necessary, synthetic, and a priori entails any particular analysis of color properties, and I doubt that the example fails. If the example should depend on a mistaken realist account of color and for that reason fail, anti-realism about shape properties is less plausible, and the proposition that nothing is round and square might serve as well. For accounts of the status of color see C.L. Hardin, *Color for Philosophers, Unweaving the Rainbow* (Indianapolis: Hackett, 1988), and Edward Wilson Averill, 'The Relational Nature of Color', *Philosophical Review* 101 (1992), 551–88. For a detailed discussion of color properties, with application to the apparently synthetic a priori proposition that nothing is red and green all over at once and with a defense of the view that color properties supervene on (and so are determined by) dispositional properties of physical objects, see Colin McGinn, 'Another Look at Color', *Journal of Philosophy* 93, 2 (1996), 537–53.

green and blue and yellow, and so on, in which we list all the remaining colors? This claim may seem right, because it *seems* self-evidently true that red is the only color filling that bill. But the claim is doubtful. For one thing, it is questionable whether a determinate list of all the other colors is even possible. More important, even if it is possible, the concept of being red is not *negative* in this way. To be red is to have *that color*; to be red is not simply to be a color other than green, blue, yellow, etc. Third, there is an important disanalogy to our paradigm of the analytic: whereas one could not have the concept of a vixen without having the concepts of a fox and a female, one could have the concept of being red (and so understand that concept) without even having all of these other color concepts (even if one must have *some* other color concept).

Moreover, proponents of the classical view would stress here (what is independently plausible) that an analysis does not merely provide a *conceptual equivalent*, that is, one which (necessarily) applies to the same things to which the concept being analyzed does, as the concept of being not-not-red applies to everything the concept of being red does. An analysis of a concept (as we shall see in Chapter 11 in exploring analyses of the concept of knowledge) must meet at least two further conditions. First, it must exhibit a suitable subset of the elements that constitute the concept; second, it must do so in such a way that one's seeing that they constitute it can yield some significant degree of *understanding* of the concept. The concept of being red is surely not constituted by the complex and mainly negative property of being a color that is not green, not blue, and so on; and one could not understand what it *is* for something to be red simply in terms of understanding that long and perhaps indefinite list.

The relevant notion of understanding is *understanding of,* not *understanding that,* which is plausibly considered a special kind of knowledge of the proposition indicated by the 'that' clause, as in the case of understanding that citizenship requires being politically informed. Understanding *of* has an intimate connection with explanation. The implication of this point here is that an analysis of a concept must provide sufficient understanding of it to provide at least some *explanation* of it. The analysis of the concept of a vixen as a female fox provides material for an elementary explanation of that concept; but noting that being red is equivalent to being non-green, nonblue, and so on for all the other colors would not provide any explanation of what it is to be red. The concept of being red is simply not thus analyzable. Even the property of being red is not identical with that negative property. Indeed, one could presumably understand the list of other colors quite well even if one had never seen or imagined redness, and one had *no* perceptual, imaginational, or other concept of redness.

The point that an analysis should provide understanding of the kind that goes with explanation must not be taken to imply that we can have understanding *only* when we can explain. It is arguable, in fact, that the concept of redness is *simple* in the sense that, unlike that of a vixen, it is not analyzable into elements of any kind. One's understanding of the concept does not require its analyzability; it is enough to be able (above all) to apply it to the right things, withhold it from the wrong ones, and see what follows from its application—such as the thing's not being green.

On balance, then, it appears that the proposition that nothing is red and green all over at once is not analytic. This does not, however, prevent our rationally grasping the truth of that proposition. Truths that meet this rational graspability condition—roughly a knowability through conceptual understanding condition— have been called *a priori propositions* (propositions knowable 'from the first'), because they have been thought to be such that they can be known a priori, in a very strict sense of this phrase: known not on the basis of sense experience but simply through reason as directed toward them and toward the concepts occurring in them, at least if reason is used extensively enough and with sufficient care. Propositions that are a priori in this strict, knowability sense—as is the proposition that nothing is red and green all over at once—are also plausibly

considered self-evident.[11] Moreover, the kind of justification for believing a selfevident proposition when we believe it in the indicated way is a basic kind of justification and is often called a priori.

By contrast with analytic propositions, however, the kind of a priori proposition exemplified by that one seems to assert something beyond what analysis of the relevant concepts can show. For this reason, propositions of this kind are also called *synthetic propositions*, though these are typically defined negatively, simply as *non-analytic*. Positively conceived, they typically bring together or "synthesize" concepts and properties, even if in a negative way (as by linking redness with colors other than green—by including it among these other colors). Synthetic propositions need not, even *in part*, analyze concepts, and many are empirical in the straightforward way in which propositions evident to the five senses are.

It is noteworthy that although analytic propositions are characterized roughly in terms of how they are *true*—by virtue of conceptual containment (or, on a related account, on pain of contradiction)—a priori propositions are characterized in terms of *how they are* known, or can be known: through the operation of reason.[12] (This allows that they can *also* be known through experience, say through receiving testimony, at least if the attester's knowledge is, directly or indirectly, grounded in the operation of reason.)

On this basis, a priori propositions are also negatively characterized as knowable "independently of experience," in which this phrase above all designates no need for evidential dependence on experiential grounds, such as those of perception. But even if this negative characterization of a priori propositions is correct so far as it goes, understanding them through it will require understanding of the kinds of positive characteristics I am stressing. Let us pursue these.

[*6. What are two kinds of objections to the idea that the proposition 'nothing is red and green all over at once' - is self-evident and necessary, yet not analytic? Briefly discuss.* 'কোনো কিছু একইসাথে ও পুরোপুরিভাবে লাল ও সবুজ নয়' – এই বচনটি সেলফ-এভিডেন্ট ও আবশ্যিক হলেও বিশ্লেষক নয়, এই ধারণার ব্যাপারে দুটি আপত্তি কী? সংক্ষেপে আলোচনা করো]

[*7. What is the relevance of 'understanding of' and 'understanding that' in the discussion of the analytic, a priori and synthetic statement?* বিশ্লেষক, অভিজ্ঞতা-পূর্ব ও সংশ্লেষক বচনের অন্তঃসম্পর্ক আলোচনায় 'understanding of' ও 'understanding that' এর প্রাসঙ্গিকতা কী?

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[11] This allows that such propositions can *also* be known empirically, say through testimony, though there are restrictions (discussed in Chapter 7) on how this may occur. The characterization suggests that an a priori proposition is knowable non-inferentially even if only on the basis of considerable reflection, but the exact mode of the appropriate reflection is not something that need be settled here. A full account of this conception of the a priori would explicate the kind of possibility of knowledge in question; it is presumably not mere logical possibility in the sense that no contradiction is formally entailed by the occurrence of the relevant knowledge, but a conceptual possibility, roughly in the sense that such knowledge is provided for by the concept of the relevant kind of knowledge: the kind grounded in understanding propositions of the sort in question. My preference is to characterize the a priori in terms of self-evident propositions and leave open what kind of possibility there has to be of the sort of understanding that grounds justification for believing those propositions. For a valuable treatment of possibility and necessity arguing that such modal notions are irreducible, see Scott A. Shalkowski, 'Conventions, Cognitivism and Necessity', *American Philosophical Quarterly* 33 (1996), 375–92.

[12] Kant's section 2b of his Preamble to the *Prolegomena to any Future Metaphysics* (trans. by Lewis White Beck, New York: Liberal Arts Press, 1950) opens with 'The Common Principle of All Analytical Judgments is the Law of [non]Contradiction' and almost immediately continues: "For the predicate of an affirmative analytical judgment is already contained in the concept of the subject, of which it cannot be denied without contradiction."

Three types of a priori propositions

If we take knowability through the use of reason as a rough indication of what constitutes the a priori in general, then it includes not only self-evident propositions but certain others that are not self-evident: most clearly those propositions *not* themselves knowable simply through reason as directed toward them and toward the concepts occurring in them, but self-evidently *following from* (entailed by) such (self-evident) propositions. This is the simplest case of what is *a priori in the broad sense*. Consider the proposition that either nothing is red and green all over at once or I am flying to the moon. This self-evidently follows from the proposition about red and green, which (apparently) is selfevident. It self-evidently *follows* because it is self-evident that *if* nothing is red and green all over at once, then *either* that is true *or* I am flying to the moon.

One might think that this disjunctive (either–or) proposition is self- evident because it is so obviously both true and necessary. But even though this is true, one knows it, not in virtue of understanding the proposition itself, but in virtue of its self-evidently following from something that *is* self-evident. One knows it inferentially, on the basis of knowing the simpler proposition that nothing is red and green all over at once. One cannot know it *just* from understanding it, as with a self-evident proposition, but only through seeing the quite different truth that if nothing is both red and green at once, then either that proposition is true or I am flying to the moon. This conditional (if–then) proposition *is* self-evident; hence, it is an utterly secure ladder on which to climb from knowledge that nothing is red and green all over at once to knowledge that either this is so or I am flying to the moon. That disjunctive proposition is *a priori in the broad sense*.

Suppose, however, that a proposition is neither self-evident nor self- evidently entailed by a self-evident proposition, but *is* provable by self-evident steps (perhaps many) from a self-evident proposition. Because there is more than one step and there can be many steps, such a provable proposition might or might not be knowable without reliance on memory, depending on the mental capacity of the rational being in question. Nonetheless, since it can be known through such a *rigorous proof*—one that begins with a self-evident proposition and proceeds only by self-evident steps (entailments) to its conclusion—a rigorously provable proposition may be called *ultimately a priori* (or ultimately self-evident, though the former term seems preferable). It is not a priori in the broad sense because (1) it is not linked to the self-evident by a *single* step and—more important—(2) it is not necessarily self-evidently linked to it.[13] But as it is ultimately traceable to a self-evident proposition, it may be considered a priori in the ultimate provability sense.

Thus, in speaking of propositions that are a priori in the most comprehensive terminology, I include not only the intuitively central cases that are self-evident or just one step from it—

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[13] There is a subtlety here that needs comment. Imagine that a self-evident axiom, *A*, self-evidently entails a theorem, *t*, which in turn self-evidently entails a second theorem, *t′*. Self-evident entailment (as opposed to entailment in general) is not *transitive*: *A* can self-evidently entail *t* and *t* can self-evidently entail *t′* without *A*'s self-evidently entailing *t′*. Here one could understand the conditional proposition that if *A* then *t′* quite adequately without thereby having justification for believing it. One might need the intermediate step, *t*, to achieve that justification, and it need not be discerned simply in adequately understanding the conditional itself. This possible limitation does not preclude there being *some* kind of understanding of that conditional and related concepts, such as a perfectly omniscient being might have, in virtue of which the proposition that if *A* then *t′* can be seen to be true. This shows that—as Aquinas saw in the quotation from him in note 9—there is a related notion—self evidence *for a particular person* (or mind)—which must be distinguished from self-evidence in its basic, non-relativized form, making reference only to anyone's understanding. Still, even if what is selfevident for God might not be self-evident for us, some propositions are unqualifiedly self-evident. The case also shows that not every proposition *provable* by individually self-evident steps from a self-evident premise may be assumed to be a priori in the (moderately) broad sense of being self-evidently entailed by a self-evident proposition; for (as just explained) such a proposition might not be self-evidently entailed by a self-evident proposition.

propositions self-evidently entailed by a self-evident proposition—but also those not thus entailed but nonetheless provable by selfevident steps from a self-evident proposition.

We could say, then, that for the kind of classical view in question, the selfevident is the *base* of the a priori: a priori propositions are those that are either self-evident (i.e., <u>a priori in the narrow sense</u>) or, though not themselves selfevident, self-evidently follow from at least one proposition that is (hence are <u>a priori in the broad sense</u>). The general notion of an a priori proposition, applicable to both cases and others, is roughly the notion of a truth that either is a self-evident proposition or is self-evidently entailed by one, or <u>provable from one by selfevident steps</u>.[14]

Knowledge of propositions a priori in the broad or ultimate provability sense, unlike knowledge of those a priori in the narrow sense, depends on knowledge of some self-evident proposition as a ground. But neither kind of knowledge depends on knowledge of any empirical proposition, and in that sense both kinds are "independent of experience."

It is because a priori propositions (of any sort) are understood in relation to how they can be *known* that the notion of the a priori is commonly considered epistemological. But many a priori propositions also have a special property of a different kind. Many are said to be analytic. The notion of the analytic is more often taken to be of a different, non-epistemological kind, say conceptual, since <u>analytic truths are conceived as grounded in a simple containment relation between concepts</u>.[15]

It should not be surprising, then, that the categories of the analytic and the a priori are not identical. In both cases, however, proponents of the classical view have taken the relevant propositions to be necessary: this is commonly thought to be obvious for the analytic ones, which are true "on pain of contradiction," <u>but it has seemed reasonable to classical theorists to hold that even synthetic a priori propositions are invariably necessary</u>. Perhaps the underlying thought is that <u>if their truth were contingent and so depended on what holds in (is contingent on) some possible situations but not others, one could not know it just on the basis of understanding the proposition itself</u>.

[*8. Discuss different types of a priori propositions. অভিজ্ঞতাপূর্ব বচনের বিভিন্ন ধরন আলোচনা করো।*

The empirical

A huge variety of truths are not a priori. That the spruce is taller than the maple is one of them. <u>Truths that are not a priori are called *empirical* (or *a posteriori*) *truths*</u>. This means, roughly, that the propositions in question can be known only *empirically*: knowable (assuming they are knowable) only on the basis of experience, as opposed to reason—above all on the basis of perceptual or selfconscious experience (in the ways described in Chapters 1, 2, and 4).

Saying simply that a proposition is empirical (or a posteriori) leaves open whether it is true: there are empirical falsehoods, such as that it is not the case that the spruce is taller than the maple, as well

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[14] In a broader usage, a falsehood can be called an a priori proposition provided it is an a priori *truth* that it is false. This less common usage raises no special problems but presents a terminological complication I ignore in the text.

[15] There is much difference in judgment about how to classify the analytic. It might be considered a semantic concept by those who think of it as truth by virtue of the *meanings* of the relevant terms. It might be regarded as ontological by those who think such truths are basic to the structure of reality. For epistemology, the notion of the a priori is the more important of the two. For an immensely influential paper arguing that neither notion is clear see W.V. Quine, 'Two Dogmas of Empiricism', in his *From a Logical Point of View* (Cambridge, MA: Harvard University Press, 1953). Among the widely noted replies is H.P. Grice and P.F. Strawson, 'In Defense of a Dogma', *Philosophical Review*
55 (1956), 114–58. For more recent discussion of these issues see Gillian Russell, *Truth by Virtue of Meaning: A Defense of the Analytic/Synthetic Distinction* (Oxford: Oxford University Press, 2008).

as empirical truths. (In this the term 'empirical proposition' is unlike 'a priori proposition' and 'necessary proposition', which are not commonly used to refer to falsehoods, but my main examples of empirical propositions will be truths.)

For the classical view, empirical propositions as well as a priori propositions are crucial for our lives. Indeed, the former include every truth known perceptually, such as those known through observing the colors and shapes of things, and all truths known scientifically, such as generalizations linking the temperatures and the volumes of gases, or ingestion of drugs with change in behavior. A certain range of a priori propositions, such as those of logic and pure mathematics, are presupposed by both common sense and science. Empirical propositions are also required to guide us in dealing with the world, but the classical view sees them as open to disconfirmation through experience in a way that a priori propositions are not.

Analytic truth, concept acquisition, and necessity

Analytic truths, as well as certain synthetic ones, are called a priori because analytic truths are knowable through the use of reason. But analytic truths appear to be knowable—or at least are showable—through a different use of reason than is appropriate to the synthetic a priori truths. It may be that I know that nothing is red and green all over at once by virtue of simply grasping, as a rational being, a kind of incompatibility between the concept of being red (at a time and place) and the concept of being green. But, as pointed out earlier, I apparently do not know it by virtue of grasping a *containment* relation between being red (or green) and anything else. If this does not illustrate two different uses of reason, it at least indicates a different kind of application of reason to different kinds of relations of concepts.

Because my knowledge of the proposition that nothing is red and green all over at once is not based on grasping a containment relation, it differs from my knowledge of the analytic truth that all vixens are female. Yet in both cases the relation between the concepts involved in the truth seems to be the basis of that truth. In both, moreover, I apparently know the truth through rationally *understanding* that relation: a relation of analytic containment in one case, and of mutual exclusion in the other.

These points do not imply that experience is irrelevant to knowledge of the a priori. On the classical view, I do need experience to *acquire* the concepts in question, for instance to acquire color concepts or the concept of a fox. But once I have the needed concepts, it is my grasp of their relations, and not whatever experience I needed to acquire the concepts, which is the *basis* of my knowledge of analytic and other a priori truths.

In part because of these similarities, as well as because the falsity of a priori propositions seems inconceivable, the classical view takes synthetic a priori truths as well as analytic truths to be necessary. They cannot be false, even though in the synthetic a priori cases it seems not to be strictly contradictory to deny one. For instance, claiming that something *is* red and green all over is not contradictory in the sense that it (formally) entails that some proposition—say, that the object in question has a definite color—is and is not true. Still, on the classical view it is absolutely impossible that something *be* red and green all over at once. We need only reflect on the relevant concepts (mainly the color concepts) to realize that nothing is red and green all over at once; we readily grasp (apprehend) an exclusion relation between being red and being green.

It is also commonly held by philosophers in the classical tradition that all necessary propositions are a priori. One rationale for this might be that necessity is grounded in relations of concepts and these (or at least the relevant relations) are the same in all possible situations. A mind that could adequately survey all possible situations (like the divine mind as often conceived) could thus know the truth of all necessarily true propositions. Since this survey method would be possible without analyzing one concept out of another, the grounding of necessity in conceptual relations would also

explain how there can be synthetic necessary truths. And for the classical view, these, being necessary, are also a priori.[16]

Summarizing, then, the classical view says that all necessary propositions are a priori and vice versa, but it maintains that analytic propositions constitute a subclass of a priori ones, since some a priori propositions are synthetic rather than analytic. The view tends to conceive the truth of all a priori propositions as grounded in relations of concepts (or of similar abstract entities, such as "universals," in Bertrand Russell's terminology).[17] But the position conceptually accounts for these propositions differently: for necessary propositions in terms of the unrestricted *circumstances* of their truth (the absolute impossibility of their falsehood in any circumstances), for analytic ones in terms of *how* they are true (typically, by virtue of containment relations), and for a priori propositions in terms of how their truth is *known* (through understanding).

[*9. The analytic propositions are known in terms of 'how' they are true and the a priori ones in terms of how their truth is 'known'. Evaluate the claim that all necessary propositions are a priori, in this regard.* বিশ্লেষক বচনগুলোকে জানা যায় 'how' they are true তার জ্ঞান দিয়ে আর অভিজ্ঞতাপূর্ব বচনগুলোকে জানা যায় how their truth is 'known- এর মাধ্যমে। এর আলোকে সকল আবশ্যিক বচনই অভিজ্ঞতাপূর্ব – এই দাবীটি মূল্যায়ন করো।]

The empiricist view of the truths of reason

The classical view of the nature of what I am calling *a priori truths—truths of reason—*and of our knowledge of them has been vigorously challenged. To appreciate the epistemological significance of reason as a source of justification and knowledge, and of truths of reason themselves, we must consider some alternative accounts of these truths.

John Stuart Mill held that ultimately there are only empirical truths and that our knowledge of them is based on experience, for instance on perception.[18] We might call this sort of view *empiricism about the* (*apparent*) *truths of reason*. The name suits the view, since the position construes apparently a priori truths as empirical, though it need not deny that reason as a capacity distinct from perception has *some* role in giving us justification and knowledge. Reason may, for example, be crucial in extending knowledge by enabling us to prove geometrical theorems from axioms. But the view I want to explore (without following Mill in particular) denies that reason grounds justification or knowledge in the non-empirical way described by the classical theory.

Rationalism and empiricism

Before we consider Mill's thesis in detail, we should contrast it, from the most general epistemological point of view, with that of Kant and other rationalists to get a better sense of what is at stake in the controversy between rationalism and empiricism. Kant's position on the truths of reason might be called rationalist, Mill's empiricist. These terms are used too variously to make precise definition wise. Very roughly, however, *rationalism* in epistemology takes reason to be far more important in grounding our knowledge than empiricism allows, and rationalists virtually always assert or imply that, in addition to knowledge of analytic truths, there is knowledge of

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[16] Critical discussion of the question whether the a priori must be necessarily true is provided in my 'Skepticism about the A Priori: Self-Evidence, Defeasibility, and *Cogito* Propositions', in John Greco (ed.), *The Oxford Handbook of Skepticism* (Oxford: Oxford University Press, 2008).

[17] See Bertrand Russell, *The Problems of Philosophy* (Oxford: Oxford University Press, 1912), Chapters 8–10 (these chapters are reprinted in Huemer, *Epistemology*).

[18] See especially J.S. Mill, *A System of Logic* (first published in 1843), particularly Book II, Chapters 5–7. For a much more sophisticated critique of apriorism in mathematics and an empiricist account of mathematical truths, see Philip Kitcher, *Mathematical Knowledge* (Oxford: Oxford University Press, 1984).

synthetic a priori truths. Very roughly, empiricism in epistemology takes experience, most notably sensory experience, to be the basis of all of our knowledge except possibly that of analytic propositions, understood as including purely logical truths, such as the truth that if all whales are mammals and no fish are mammals then no whales are fish. (For both empiricists and rationalists, analytic propositions are typically taken to include logical truths.)[19]

One might wonder why some empiricists grant that analytic truths may be a priori. The central point (though an empiricist might not put it this way) may be seen if we use the terminology of the classical theory: even if such logical propositions are not true by virtue of containment relations between concepts, their negations formally entail contradictions, for instance that some vixens are and are not female foxes. They are therefore paradigms of truths of reason; for the use of logic alone, which is perhaps the purest use of reason, can show that they can be false only if a contradiction is true—which is absolutely impossible. This is another reason why, as noted above, analytic propositions are sometimes given a broader characterization than I have proposed and are taken to be those whose negations entail a contradiction.[20]

Some empiricists do not allow that any knowledge, even of so-called analytic propositions, is genuinely a priori. A *radical empiricist*, such as Mill, takes *all* knowledge to be grounded in experience. A *radical rationalist* (which Kant was not) would take all knowledge to be grounded in reason, for instance to be intuitively grounded in a grasp of self-evident propositions or deductively based on inference from a priori truths that are intuited.[21]

[*10. What do you know 'paradigms of truths of reason'? Do you support radical rationalism, or radical empiricism? Put arguments in your favor.* বৌদ্ধিক সত্যতা সম্পর্কিত চিন্তাকাঠামো সম্পর্কে কী জানো? তুমি কি চরম বুদ্ধিবাদ সমর্থন করো, অথবা চরম অভিজ্ঞতাবাদ? তোমার পক্ষে যুক্তি দাও।]

Empiricism and the genesis and confirmation of arithmetic beliefs

Empiricism about what are called the truths of reason is most plausible for the apparently synthetic a priori ones, so let us sketch it with reference to an apparently synthetic kind of a priori proposition that has been much in dispute. Mathematical truths, particularly truths of simple arithmetic, are often regarded as synthetic a priori. Consider the proposition that $7 + 5 = 12$ (Kant's example, also found in Plato's *Theaetetus*). It is easy to say that one just knows this, as one knows that nothing is red and green all over at once. But how does one know it?

Here we cannot readily find a good analogy for the simple exclusion relation we apparently grasp in the case of red and green. Could it be that from experience with objects, say with counting apples, then combining two sets of them, and recounting, we learn our first arithmetic truths and then use reason to formulate general rules, such as those for calculating larger sums?

Viewed in this way, arithmetic develops rather as scientific hypotheses are often thought to, with observations crucial at the base, generalizations formulated to account for them, and broader

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[19] Granting it is at best not obvious how logical truths are knowable by any analysis that reveals containment relations, their negations can be clearly seen to entail contradictions.

[20] How broad this is depends on the notion of entailment used. I have in mind a notion for which the negation of a proposition entails a contradiction provided that the use of formal logic, supplemented only by (correct) definitions, renders a contradiction deducible.

[21] Someone might think all truth is a priori on the ground that it is true a priori that (1) God exists; (2) a certain universe specifiable in every detail is the best of all possible universes; and (3) God creates the best of these universes. Then, with sufficient intellectual power, one could (arguably) reason one's way to any truth. Gottfried Wilhelm Leibniz (1646–1716) has been read as holding a view close to this (but there are reasons to doubt that he did, including considerations about divine freedom).

generalizations postulated to link all the observations and the narrower generalizations together. And do we not first learn to add by counting physical things, or by counting on our fingers?[22]

To be sure, we perhaps cannot imagine how the number 7 added to the number 5 could fail to equal the number 12. But that is not a point about the behavior of objects in the physical world. The physical world *could* go haywire so that when (for instance) five apples and seven oranges are physically combined, the result of counting the new set is *always* eleven. If that happened pervasively, might we not begin to think that arithmetic must be revised, just as Einstein's work showed that the physics of "the incomparable Sir Isaac Newton" needed revision? Perhaps the crucial epistemological consideration is what overall account of our experience is most reasonable; and if the best overall account should require rejecting a proposition now considered a priori and necessary, so be it.

The classical view provides for several critical responses. One concerns the distinction between two quite different things: the *genesis* of one's beliefs—what produces them—and their *justification*, in the sense of what justifies them. A second point concerns whether arithmetical propositions can be tested observationally. A third focuses on the possibility of taking account of what looks like evidence against arithmetical truths, so that even if one's final epistemological standard for judging a proposition *is* its serving the demands of the best overall account of experience, these truths can be preserved in *any* adequate account. Consider these ideas in turn.

First, granting for the sake of argument that our arithmetic beliefs arise from counting physical objects, is the experience that produces them what *justifies* them? The genesis of a belief—what produces it—is often different from what justifies it. The testimony of someone I realize is unreliable might, when I am off guard, produce my belief that different brands of aspirin do not, apart from additives, differ chemically. My belief would at that point be unjustified; but it might become justified later when I learn that aspirin is simply acetylsalicylic acid. Moreover, regardless of what produces our arithmetic beliefs initially, when they are justified in the *way* my belief that $7 + 5 = 12$ now is, experience does not appear to be what justifies them. For my part, I do not see precisely how the truth of the proposition might be grounded in the behavior of objects when they are combined; and I would not try to justify it, as opposed to *illustrating* it, by citing such behavior.

This brings us to the second point: it is doubtful that the proposition that $7 + 5 = 12$ is (empirically) *testable*, say by examining how objects combine, though it is *exemplifiable* in that way. The empiricist might reply that this by no means shows that the proposition is, as the classical view insists, necessarily true rather than contingent and empirical. Indeed, it does not. But let us look closely at the idea that it could be tested, and could thereby be disconfirmed by discovering that when groups of five objects are combined with groups of seven, we find just eleven.

This brings us to a third response. How might one deal with repeated and systematic counter-evidence? Classical theorists will argue that it is possible for the world to alter in such a way that this combination procedure results in one item's disappearing, or in our failing to see it, or in our misremembering how many items entered the mix before our re-counting. They will also argue that the unexpected realization of such possibilities would be a better interpretation of the strange cases described—hence of our overall experience—than saying that it has turned out to be false that $7 + 5 = 12$. Thus, instead of saying that an arithmetical principle has been falsified, we would say that the world no longer uniformly exemplifies it.

One consideration favoring the classical view is that it is at best difficult even to understand how the purely arithmetical principle could be false. The number 7 plus the number 5 apparently equals the number 12, regardless of how apples and oranges behave. For the arithmetic statement is

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[22] Cf. W.D. Ross, explicating how Aristotelian *intuitive induction* can yield a priori knowledge: "We find by experience that this couple of matches and that couple make four matches . . . and by reflection on these and similar discoveries we come to see that *it is of the nature of two and two to make four*" (*The Right and the Good* [Oxford: Oxford University Press, 1930], p. 32).

apparently not *about* apples and oranges, though (so far as we know) their behavior exemplifies it. For the classical view, at least, it is about *numbers*, ~~which, unlike the Arabic or Roman or other numerals we use to represent them linguistically~~, are abstract and non-physical. If a proposition is not about concrete objects, facts about their behavior are not a test of its truth.

Notice something else. In order to gather purportedly significant counterevidence to the arithmetic proposition in question, one would have to rely, as already noted, not only on memory and perception (both highly fallible sources) but also on simple arithmetic: one would have to *count* disconfirming cases. A single apparent instance, say of seven and five things brought together and not adding up to twelve, would not be significant, and one must keep track of how many anomalies there are, relative to confirmatory instances in which the expected sum is counted out. It is not normally reasonable to give up a good theory on discovering a single apparent counter instance. It appears, then, that in order to take seriously empirical evidence that would undermine arithmetic, we must trust perception in our counting, arithmetic itself in our summing, and memory in our overall judgment.

One might think it is enough simply to *have* a significant number of such disconfirming cases. But this is not so. One must be justified in believing that the number *is* significant. And how could one achieve this if one either made no count or—in any case—could not rely on one's count of single cases to sum to a significantly large number? If it need not be true that $7 + 5 = 12$, why should $1 + 1 + 1$ disconfirming instances necessarily sum to 3? And would anything less than a huge number of apparently disconfirming cases be evidentially decisive against such a proposition of simple arithmetic? A single disconfirming instance would surely seem just an anomaly; there must be a significant number. One would, then, have to rely on some arithmetic propositions, such as that $1 + 1 + 1$ disconfirmations = 3 (a minimally significant number, perhaps), in order to mount an effective challenge to the (necessary) truth that $7 + 5 = 12$. Given the interconnections among arithmetic propositions, it is not clear that one could consistently (or at least with any plausibility) maintain the needed disconfirmatory propositions while denying that $7 + 5 = 12$. Still another obstacle to recognizing apparent counter-evidence as genuine is the dependence on memory to keep track of disconfirming instances. The fallibility of memory would defeat confidence that one had adequately tracked apparent disconfirmations.

There may be a way around these objections, but even finding it would leave one far from a strong case for the contingent or empirical status of arithmetic truths.[23] Even if one appealed, not to apparent counter-instances to the proposition that $7 + 5 = 12$, but to a well-confirmed *theory* to argue that it could be false, one would need to do at least some counting of one's confirmatory data regarding that theory (not to mention other ways in which theory confirmation relies on arithmetic, perception, and memory).

None of these points requires us to deny that there *is* a similar, contingent arithmetic proposition about apples and oranges, namely that when we *count* five of the first and *place* them next to the result of counting seven of the second, we *can count* twelve all told. This proposition may easily be confused with its pure mathematical counterpart. The former is clearly contingent and empirical, but its being so does not show that the purely arithmetic proposition is also. The distinction between pure and applied mathematics can also be brought to bear on geometry.[24]

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[23] The proposition that $1 + 1 + 1 = 3$ might be held to be more intuitive than the proposition that $7 + 5 = 12$. But, first, in practice we might need to rely on less intuitive or much more complicated arithmetic to get a good case for the possible falsehood of the original proposition; second, and more important, the simpler proposition that $1 + 1 + 1 = 3$ will also do as a case of a necessary mathematical truth.

[24] For discussion of the status of the a priori in connection with geometry, see the Appendix to Laurence BonJour, *In Defense of Pure Reason* (Cambridge: Cambridge University Press, 1998). That book is also of interest for its criticism of Kant, who in BonJour's view is less a rationalist about—and less plausible concerning—the a priori than is often thought.

There is a related metaphysical dimension of the question of the status of arithmetic truths. By contrast with at least one form of the classical view, radical empiricism denies that there *are* abstract entities and so, believing that mathematical propositions are about something concrete, radical empiricists naturally view them as generalizations about the behavior of physical objects. We need not accept the empiricist view to grant that if physical things did not exemplify the proposition that 7 + 5 = 12, the proposition would be of far less *value* to us even if necessarily true. If the physical world went haywire, it could turn out to be false that when seven apples are placed together with five more and the total collection is counted, the count yields twelve. This chaotic situation would falsify the *physical* principle already contrasted with the arithmetic one in question. But the physical principle is not, and does not even follow from, the purely mathematical proposition we are discussing.

[*11. How do the empiricists accommodate the genesis and confirmation of arithmetic beliefs? Why the arithmetic 'truths' are written as arithmetic 'beliefs' in the texts on epistemology, anyway?* অভিজ্ঞতাবাদীগণ কীভাবে গাণিতিক বিশ্বাসসমূহের উদ্ভব ও নিশ্চয়তাকে ব্যাখ্যা করে? আচ্ছা, জ্ঞানতত্ত্বের ওপর যে সব পাঠ্যবই রয়েছে তাতে গাণিতিক 'সত্যসমূহ'কে গাণিতিক 'বিশ্বাস' হিসাবে লেখা হয় কেন?]

Empiricism and logical and analytic truths

The empiricist view of the a priori can also be applied to analytic propositions and even to self-evident logical truths, and it may indeed appear more plausible in that case. Suppose that through scientific investigation we discover that vixens have certain characteristics we think of as male, such as certain hormones. Imagine that gradually (perhaps because of chemicals in the environment) these discoveries mount up so that the female foxes in our laboratory begin to seem more aptly classified as male than as female. Could not a time come when we begin to doubt that vixens are female after all?

And what about the logical principle of the excluded middle, which says that every proposition is either true or false? Consider the proposition that Tom is bald. Must this proposition be either true or false no matter what the quantity or distribution of hair on his head? Surely the proposition is an appropriate counterexample to the principle of the excluded middle.[25]

The classical view can offer its own account of these examples. For one thing, particularly over a long time, we can begin to use a term in a sense different from the one it now has. Thus, the discoveries about vixens could result in our someday using 'vixen' to mean not 'female fox', but 'fox with female external sexual characteristics and of the anatomical kind *K*' (where *K* is the kind of animal we have in our laboratory). Then, when we utter such words as 'Vixens are not really female', we are not denying the analytic proposition *now* expressed by 'All vixens are female'. We have confirmed something else, rather than disconfirming this.

In this way, then, our experience might result in our someday no longer assertively uttering 'Vixens are female' to say anything that we believe. This certainly does not show that experience might falsify the proposition we *now* affirm when we assertively utter that. Given what we now mean by 'vixen', in saying that all vixens are female we do not rule out that *those* 'vixens' in the lab could have internal biological and chemical characteristics in the light of which *they* ultimately need not be considered female.

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[25] For discussion of vagueness and its bearing on epistemological matters (as well as references to his own and others' earlier work on vagueness) see Timothy Williamson, *Vagueness* (London: Routledge, 1994) and *Knowledge and its Limits* (Oxford: Oxford University Press, 2000).

Regarding the <u>principle of the excluded middle</u>, I would stress that Aristotle plausibly argued against it, and some contemporary philosophers of logic do, too. The main reasons for doubting it, moreover, do not depend on empiricism. Let us explore some of them.

Consider again the vague statement that Tom is bald. It may certainly be argued that this need not be either true or false. It is not as if 'bald' meant, say, 'having fewer than 500 hairs on the top of one's head'. It does not. And if it did, the term 'top' would still be vague and would cause the same trouble: it would be unclear in what *area* we must find 500 hairs. If the middle possibility—neither truth nor falsity—is to be ruled out, it must be by a better argument. The principle of the excluded middle, though often used to suggest that even logical truths are not necessarily true, is controversial among rationalists and empiricists alike. The principle is a poor example to support the empiricist case against the necessity of logical truths.

When, by contrast, standard examples of <u>simple logical truths</u> are used, the effect seems very different. Consider the proposition that if Ann is coming by bus or she is coming by plane, and it is false that she is coming by bus, then she is coming by plane (which exemplifies the general logical truth that if at least one of two propositions is the case and the first is false, then the second is true). Is there any plausibility in the view that this might be false? I find none; and while nothing said here proves that the empiricist <u>account of the a priori</u> is mistaken, it appears less plausible than the <u>classical account</u>.

If what we have seen so far is accepted, the classical view of the truths of reason is quite defensible and the empiricist critique of it fails. But we have not yet adequately taken into account the ways in which knowledge of those truths might depend on language. This is an important topic particularly given the extent to which understanding the a priori is connected with understanding language. The next chapter will consider this topic in some detail.

[*12. **How do the empiricists accommodate the logical and analytic truths? Critically explain.***
অভিজ্ঞতাবাদীগণ কীভাবে যুক্তিবিদ্যার নিয়মাবলী ও বিশ্লেষনী সত্যসমূহকে গণ্য করে? সমালোচনামূলক ব্যাখ্যা দাও।]