Some Reflections on the Ontology of Zero

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Is zero nothing, or something? This is the central question of the present study.

Structure of the Study:

1. Relevance of the study: Numbers as metaphysical entities.
2. Key points of discussion
3. Philosophical implications of the study
4. Primary knowledge of math that is necessary for following this issue.

Key points of discussion

1. Zero as nothing in the non-existing sense and zero as apparently 'something' existent sense:

Zero as nothing:

0+0=0 0-0=0 0-1=-1 1+0=1 1-0=1 0×0=0 0×a=0

0 is nothing in **Fibonacci series** of numbers. 0+0=0, 0+1=1, 1+1=2, 1+2=3, 2+3=5, 3+5=8, 5+8=13 …

Zero as something:

0/0=indeterminate 1/0=undefined 1×0=0 0 as an even number

0 divided by 0 could be 0,1,2,3,4, … So, it is considered that zero divided by zero is indeterminate.

1 divided by 0 could be +∞ or -∞, so it is considered that 1 divided by 0 is undefined.

And, any number multiplied by 0, will result 0.

0 has all the qualities of an even number.

Given **set theory**, we can generate an infinite number of sets out of just an empty set or null set.

{0} ≠ Ø. Because, 0 is not an empty set. 0 is something. **n(Ø)=0**, i.e. number of members in an empty set is zero.

2. Near to 0 vs absolute 0; and close to infinity versus infinity itself:

In case of 1 divided by 0 (1/0), if 0 is not approached from +1 or -1, then a real number, 1 for example, subtracted by nothing, could result both 0 or 1 from different perspectives.

From **result-oriented arithmetic** O. If I have 1 thing and I don't give you any part of it to you, then you have got nothing. So, the result is 0 from your perspective.

and from **commonsense perspective**, the result would be 1. The rationale of the commonsense perspective is something like this: if something is subtracted by nothing, that subtraction will be meaningless. if so, then that subtraction will make no change and have no impact on that something, 1 in this case. If I have 1 thing, and if I don't give it to you then I continued to have 1 thing in either situation.

If 1 is divided by 0, and if 0 is taken or approached from 1, then the division will be infinitely smaller (fractions of 1). Necessarily, the result will be proportionately ever bigger. In such a situation, mathematicians consider 'near to zero' as zero; and 'near to infinity' as infinity. On basis of this, they claim that 1 divided by 0 is ∞ (1/0=∞). When accurate result is not possible or attainable, mathematicians use 'limit'. Suppose, 4.99999 …'s limit is -5 and 5.0000…1's limit is +5

Problems of this assumption:

(1) near to 0 ≠ 0; and, near to ∞ is ≠ ∞
(2) How could we be sure about any specific number as 'near to' zero or infinity?
(3) We can't divide any physical object or phenomenon beyond it's limit.

Instead of applying commonsense, if we take mathematical findings and accuracy seriously, so many physically real things will be impossible and unreal (example, Zeno's paradox); and so many impossible entities may appear as possible (example, a circle with zero or minus radius).

3. Commutative property of numerical interchangeability and applicability of relational transitivity

Mathematicians claim that a×0=0 on basis of a×b=b×a and a+b=b+a

mathematicians propose it in the following way:

if a×b=b×a, then applying the commutative property of multiplication, we can put it in this way: a×0=0×a. we have already agreed that 0×a=0. so, it's easy, a×0=0

But we know, only counting numbers could have this kind of commutative property. Beyond this mere numerical similarity, transitivity claims are very crucial. Relations are sometimes transitive, sometimes not. Mutual value shifting, which we call transitivity, is grounded in and determined by commonsense reality, not by mathematical reality.

In **Matrix** (matrix is a number system or array of numbers) AB ≠ BA.

In **Cartesian Coordinate** System (2,3) and (3,2) are different points, though the amount of area is same.

Moreover, of null sets, **Zermelo reduction** is different in **Von Neumann reduction** from 2nd, 3rd, 4th … sub-sets of the original null set.

eat≠ate, though these two words have the same alphabets. These two words comprise the same set of alphabets, yet they are different words, not identical with one another.

Identity is more than similarity, equivalence, implication or even necessity. Issues like multiple realizability problem and end-means debate are very much relevant here.

Mathematicians sometimes 'prove' something in one way and then apply that very specific 'proof' in all the similar cases. This approach actually breaches the **falsifiability principle**.

If something is a 'proof' of anything, then that 'proof' must retain its validity in all the similar cases. If it is not, i.e. if it is true in some relevant cases and not in some another but quite relevant cases, then it is not a valid law. And we know, exceptional 'proof' is not proof and exemplary majority do not mean or make a scientific law.

4. Zero's affinity with points and infinity:

Infinity is not a number, though the total number system is, in a sense, based on it. Without infinity we can't think about the very number line. This non-numerical but conceptual mathematical entity is quite mysterious and obvious.

Contrary to ꝏ, 0 is a number. But it's a unique number. Each and every number has a value, else zero. Without any independent value, it's just a divider of positive and negative numbers.

Like negative and imaginary numbers, it is a real assumption. It is like our non-physical but true imaginations.

0 is like point. It has a position, the in-between position of a number line. As the physical pointlessness of a point, zero also don't have any numerical value, specifically when we consider zero as nothing, meaning non-existence or total absence.

Surprisingly, this very nothingness plays vital role. Sometimes as nothing and sometimes as 'something'.

At this level, from this regard we can say, in a sense, **0 is a numerical form of ꝏ**, though zero and infinity is not identical (0≠ꝏ). They have common characteristics.

In our observation, **0 is the cross-point of infinites in either directions**. We assume ꝏ but can't calculate it. We assume 0 and we can use it in our calculations.

0-as-nothing is physically nonexistent. In that sense, **0 is a pragmatic ontological concept**.

On the contrary, zero-as-something is meant something existent. In this sense, **0 is a neutral, equilibrium or balancing factor**.

5. 0’s multiplicative inverse:

A number must have its multiplicative inverse. How can there be a number without multiplicative inverse? Yes, 0 is number without any multiplicative inverse. So, it’s an exception or a contradiction. In this sense, 0 is more like a concept than a number. In this specific context, 0 is considered as nothing in the non-existent sense.

According to the mathematicians, product of any number with 0 is 0, i.e., $1\times0=0$. So, there can’t be any number which is multiplied by zero, will produce 1. In this sense, 0 has no multiplicative inverse. Remember, any number, multiplied by its multiplicative inverse, the outcome or result has to be always 1.

But if we consider the commonsense view and logic as the ultimate criteria of all human knowledge including mathematics, then we could easily find that $\frac{1}{0}$ could easily be the multiplicative inverse of 0. In that case, the math will be $\frac{0}{1} \times \frac{1}{0} = 1$

6. Observer-dependency and context-sensitivity of mathematics:

Math is highly context-sensitive. It is always conditioned by taken or given or for granted factors, which they call axioms. In other words, Mathematical truths are simply logical deductions of given or taken definitions and axioms. These definitions and axioms are metaphysically subjective, epistemologically objective.

According to Gödel’s incompleteness theorem, “in any language expressive enough to describe the properties of the natural numbers, there are true statements that are undecidable in the sense that their truth cannot be established by any algorithm.”

A few philosophical implications of this study:

1. Kant and arithmetic statement:

According to Kant, arithmetic statements are examples of synthetic a priori. If 0 is a number, no matter it is nothing or something, Kant is wrong.

2. Krauss’s ‘A Universe from Nothing’:

Lawrence Krauss is a renowned astrophysicist. According to him, everything has been created from nothing. If we consider zero, as an equilibrium 0 or balancing 0, then that 0 is a meaningful 'something'. If so, then Krauss is wrong.

3. Cognitive centrality:

cognitive centrality is a combination of metaphysical subjectivity and cognitive objectivity. 0 is metaphysically subjective but epistemologically objective. Cognitive centrality is explainable both from 1^{st} and 0^{th} point of view.

4. Ontological dualism:

If mathematical reality is valid or a 'real' form of reality, then ontological dualism is correct. Zero and negative numbers are 'real' but not physical.

5. Commonsense and science:

Commonsense is based on physical senses, but occasionally it goes beyond the boundary of physicality. No science, actually nothing can cancel commonsense, but improve its quality. In other words, common sense is the base of all sorts of human knowledge including mathematics.

6. Mathematics and Philosophy:

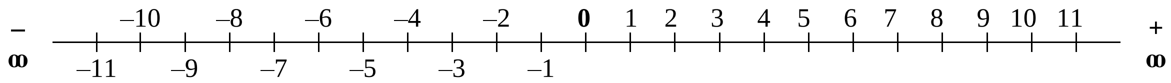
Math is a tool based on philosophical assumptions. Without relevant philosophical support, math is contradictory, meaningless and impossible. The so-called pure (or, puritanic?) mathematical reality leads us to philosophical skepticism, which is self-contradictory and self-refuting from epistemic point of view.

Necessary Primary Knowledge of Math

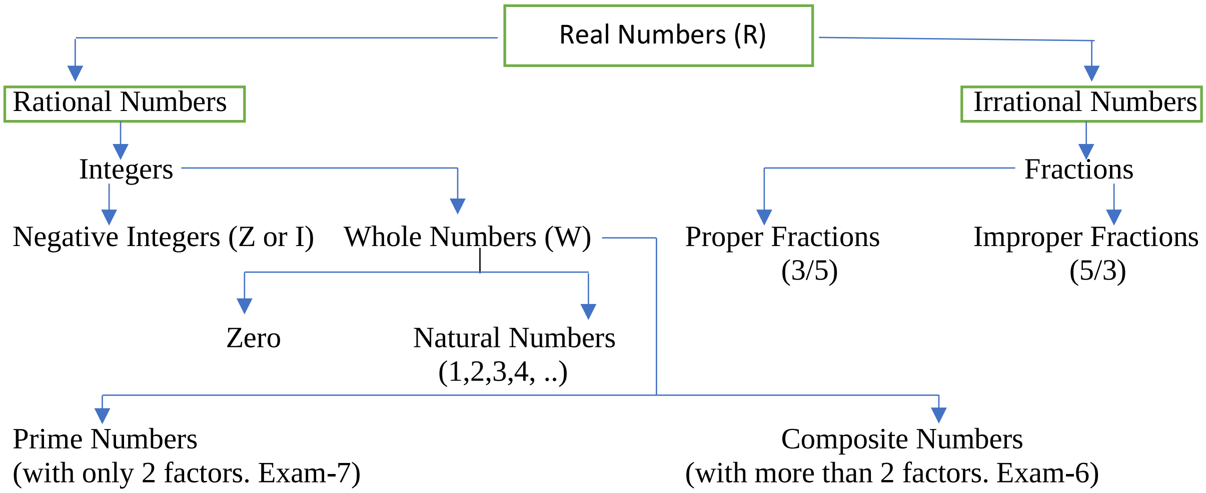
Few Number Systems

decimal	0	1	2	3	4	5	6	7	8	9	10	11	12
binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100
octal	0	1	2	3	4	5	6	7	10	11	12	13	14
hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C

Real number line



Classification of Numbers



$$\boldsymbol{Pi\ \pi} = 3.14159265359\ldots = \frac{Circumference}{double\ radius\ (2\ r)\ \text{or}, diameter}$$

Fibonacci number Series (F_n): 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, … the golden ratio φ 1.618…

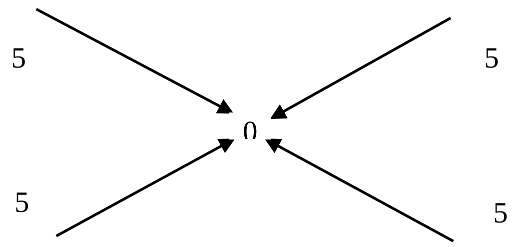
Multiplication is a short form of addition: $2 \times 3 = 2 + 2 + 2 = 6$

Division is a short form of subtraction: $\frac{6}{2} = 6 - 2 = 4 - 2 = 2 - 2 = 0$

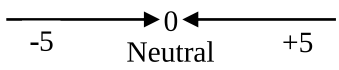
Zermelo Reduction of empty set
0 = Ø
1 = {Ø}
2 = {{Ø}}
3 = {{{Ø}}}
4 = {{{{Ø}}}}

Von Neumann Reduction of empty set
0 = Ø
1 = {Ø}
2 = {Ø,{Ø}}
3 = {Ø,{Ø},{Ø,{Ø}}}
4 = {Ø,{Ø},{Ø,{Ø}},{Ø,{Ø},{Ø,{Ø}}}}

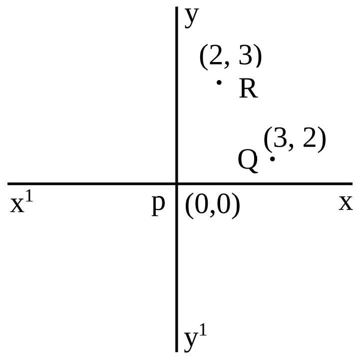
Equilibrium:



counter balanced:



Cartesian coordinate system:



Multiplicative Inverse:

$\frac{10}{2} = 5$ $\quad 5 \times 2 = 10$ here the multiplicative inverse is $\frac{1}{2}$ of the denominator 2

$2 \times 5 = 10$ multiplicative inverse of the denominator 5, is $\frac{1}{5}$

$3 \times 2 = 6$ $\quad 6 \times \frac{1}{2} = 3$ We see that $\frac{1}{2}$ is the multiplicative inverse of 2

any number, multiplied by its multiplicative inverse, result will be always 1 (one).